MR1788996 (2002f:01002) 01A16 (01A55 01A70 51-03)

Herz-Fischler, Roger

★The shape of the Great Pyramid. (English summary)

Wilfrid Laurier University Press, Waterloo, ON, 2000. *xii*+293 *pp*. $29.95. *ISBN* 0-88920-324-5

This book is a gift to everybody who is regularly exposed to pyramidological questions and wants to give a decent and reasoned answer without spending months or years on the topic. It is also of great help to anybody who is interested in knowing for one or the other reason how precisely the dimensions and appurtenant angles of the Great Pyramid can be known; finally, it is interesting for the insights it offers into the sociology of knowledge of Victorian (and not only Victorian) science and popular science.

A first, short part of the book (expanded in a number of appendices) presents the framework for the discussion: the historical and architectural context of the Great Pyramid; its external dimensions (the internal structure with its chambers and galleries is not dealt with) and the basic ancient Egyptian metrologies of relevance either for the construction or for later discussions; and a historiographical overview reaching from Herodotus's *Histories* to Borchardt's *Gegen die Zahlenmystik an der grossen Pyramide bei Gise* from 1922.

The bulk of the book is constituted by Part II, "One pyramid, many theories". Here, a number of theories are presented: Arris (edge from top to corner of base) = Side (of base); Side = Apothem (distance from top to mid-point of side); Side:Height = $8{:}\,5$; "π theory": $4 \times \mathrm{Side} = 2\pi \times \mathrm{Height}$ (square base obtained by "squaring" the perimeter—not the area—of a circle with radius equal to the height); "Heptagon theory": angle of declination of the face $= 360°/7$; "Kepler triangle theory": a cross-section parallel to the side consists of two "Kepler triangles", right triangles whose sides are in continuous proportion (whence the ratio of the apothem to the halved side is the "Golden Number" $(1+\sqrt{5})/2$); Side:Height = Golden Number; "equal area theory" (namely, equality of the triangular face and the squared height) (it is mathematically equivalent to the Kepler triangle theory, but this was not observed when the theories were first presented); Slope of the Arris = 9/10; Height:Arris = $2{:}\,3$. All of these are discussed in the chronological order in which they were invented. At first, however, the "seked theory" is introduced, the only theory which is corroborated by an ancient Egyptian mathematical text (the Rhind Mathematical Papyrus). It states that the Pyramid was constructed with a seked (the run corresponding to a rise of 1 cubit) of 5 hands, 2 fingers (which is mathematically identical with the assumption Side:Height = $11{:}\,7$, the prediction of the π theory if π is approximated as $22/7$).

The mathematical implications of all the theories are worked out in detail and compared to the measured values (those of today as well as those used at the time); after that the career of each theory is followed, from the original formulation (or precursors who came close or spurred it) onward, with a precise description of further proponents of importance, including analysis of their general orientation, motivations and mutual relations. The seked theory is also correlated with the actual (meagre) archaeological evidence for the actual techniques used to determine slopes.

Part III is divided into three sections. The first is a very perspicacious discussion of the criteria which must be applied when judging theories like the ones discussed in the preceding section. None of them are astounding, but the reviewer does not remember having seen them put together in this systematic way and therefore finds that they deserve to be listed (much paper might have been saved if pyramidologists had applied them, however permissively): (1) A theory must correspond to a level of mathematics consistent with what was known to the ancient Egyptians. (2) A theory must correspond to the accuracy of Egyptian calculations. (3) For a theory to be credible some reason must be given as to why anyone would use such a criterion in designing a pyramid. (4) To be credible a design principle should be applicable to other pyramids. (5) To be credible, a design principle should be capable of being translated with as little manipulation as possible into a practical method of construction. That such criteria are needed is demonstrated by an exquisite discussion of how densely seemingly significant mathematical relations fall around the actual slope of the Great Pyramid.

The next section, “Sociology of theories. A case study: the pi-theory”, investigates the fate of this theory in Victorian Britain, and concludes, from an analysis of the attitudes of its proponents, their other writings and the publishing channels they used, that its success was linked to anti-Darwinism, to religious fundamentalism, and to anti-French opposition to the metric system.

The last section of Part III summarizes the conclusions, both on the mechanisms of the sociology of theories and on the design principle for the Pyramid. On the latter account it is pointed out that we do not know with what precision all faces had the same slope, since all precise measurements are made on the north side, and that even the seked and the run-to-rise theories are not fully convincing (the fractions of fingers of which some of the Rhind Papyrus problems make use for the seked would be inconvenient in practical construction).

The book is well written in a pleasant personal style; the argument is always clear and lavishly supported by notes. The extensive bibliography indicates in which libraries the often rare items can be found and in which place each work is referred to.

The reviewer would add one minor correction: Kepler was not the first to write about the “Kepler triangle”, as asserted on p. 229, note 3. Formulated in terms of a rectangle with diagonal, it occurs as Problem 51 in Abû Bekr’s “Liber mensurationum” [J. des Savants **1968**, avril-mai, 65–124], and again in L. Fibonacci’s “La practica geometriae” [in *Scritti di Leonardo Pisano, Vol. II*, Boncompagni, Rome, 1862] in words that differ from Abû Bekr’s and with a method that comes close to what Herz-Fischler reports about Kepler’s treatment (division of the hypotenuse in extreme and mean ratio). It is also found (dealing with a triangle) in P. Nunes’ *Libro de algebra en arithmetica y geometria* [Stelsio, Antwerp, 1567 (fol. 248v, arithmetical solution fol. 201f–203v)] but treated rather differently; it is thus likely to have been widespread in late medieval and Renaissance manuscript traditions.

{Reviewer’s remark: A number of photos, diagrams etc. discussed in the book but not included in it for financial reasons have been posted on the Wilfrid Laurier University Press website, www.wlu.ca/~wwwpress/wlup.html. Simply click on “Books” and then search by author or title.}

Reviewed by *Jens Høyrup*